EQUATIONS OF LINES

1 Find the equation of the straight line which passes through the point $(-1,2)$ and is:

(a) parallel to the line with equation $x = 2$

(b) perpendicular to the line with equation $y + 3x = 0$

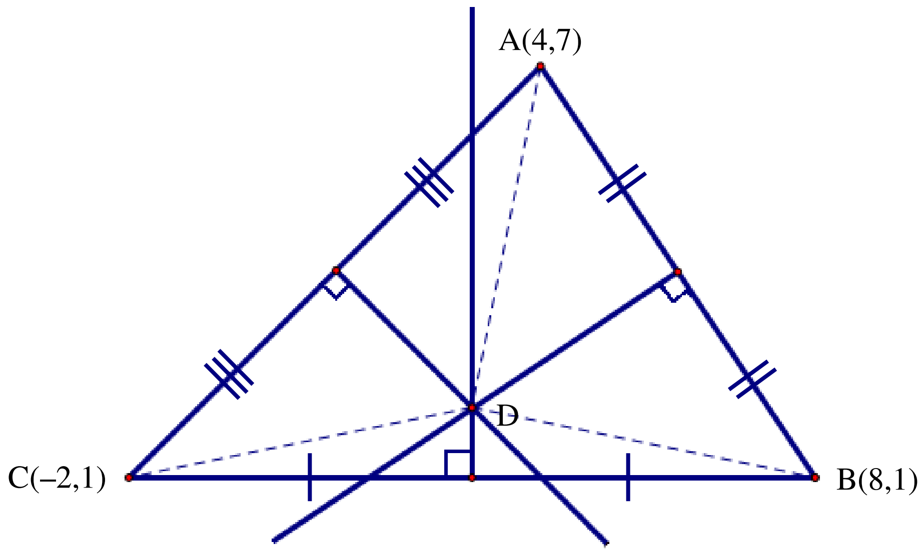
(c) parallel to the line with equation $y - \frac{2}{3}x = 4$

2 Find the equation of the perpendicular bisector of the line joining P(2,3) and Q(8,–1).

3 Find the equation of the median AD of the triangle ABC where the coordinates of A, B and C are (–3,2), (–4,–3) and (4,1) respectively.

4 D(–2,6), E(0,–3) and F(11,2) are the vertices of a triangle DEF.
Find the equation of FG, the altitude from F to DE.

5 The perpendicular bisectors of the sides of a triangle are concurrent at a point which is equidistant from the vertices. i.e. AD = BD = CD. This point is called the circumcentre.



By solving the equations of two of the perpendicular bisectors, determine the coordinates of point D the circumcentre of triangle ABC.

1a) Line $x = 2$ is vertical

$\Rightarrow$ $\parallel$ line through $(-1, 2)$ is $x = -1$

b) $y + 3x = 0$

$y = -3x$

$m_1 = -3$

$\Rightarrow m_2 = \frac{1}{3}$ as $m_1 . m_2 = -1$ for $\perp$ lines

$y - b = m(x - a)$

$y - 2 = \frac{1}{3}(x + 1)$

$3y - 6 = x + 1$

$x - 3y + 7 = 0$ is $\perp$ to $y + 3x = 0$ through $(-1, 2)$

c) $y - \frac{2}{3}x = 4$

$y = \frac{2}{3}x + 4$

$m_2 = m_1 = \frac{2}{3}$

$y - b = m(x - a)$

$y - 2 = \frac{2}{3}(x + 1)$

$3y - 6 = 2x + 2$

$2x - 3y + 8 = 0$ is $\parallel$ to $y - \frac{2}{3}x = 4$ through $(-1, 2)$

2 $P(2,3)$ $Q(8,-1)$

Midpoint is $(5,1)$

$$m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-1-3}{8-2}$$

$$= \frac{-4}{6}$$

$$m_{PQ} = -\frac{2}{3}$$

$$\Rightarrow m = \frac{3}{2}$$

as $m_1 . m_2 = -1$ for $\perp$ lines

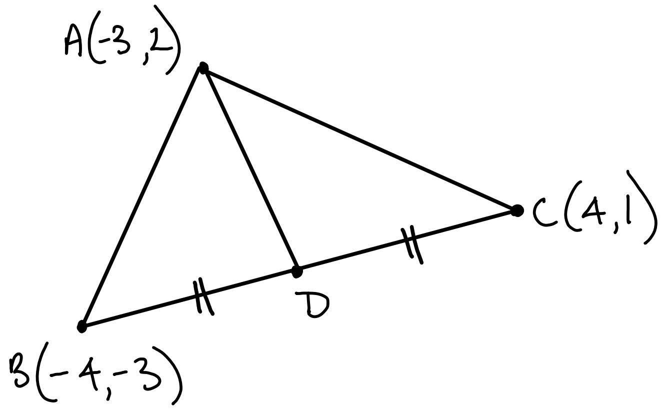
$$y - b = m(x-a)$$

$$y - 1 = \frac{3}{2}(x-5)$$

$$2y - 2 = 3x - 15$$

$3x - 2y - 13 = 0$ is equation of perpendicular bisector of PQ

3.



$D(0,-1)$

$$m_{AD} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-1-2}{0-(-3)}$$

$$= \frac{-3}{3}$$

$$m_{AD} = -1$$

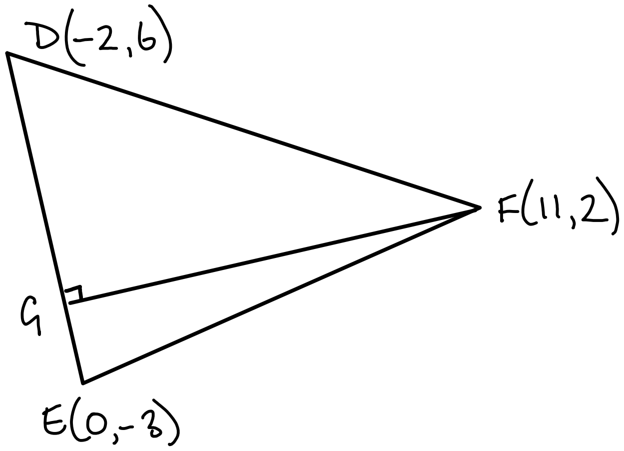
$$y - b = m(x-a)$$

$$y - 2 = -1(x+3)$$

$$y - 2 = -x - 3$$

$x + y + 1 = 0$ is equation of median AD

4.



$$m_{DE} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-3-6}{0-(-2)}$$

$$m_{DE} = \frac{-9}{2}$$

$\Rightarrow \quad m_{FG} = \frac{2}{9}$ as $m_1 . m_2 = -1$

for $\perp$ lines

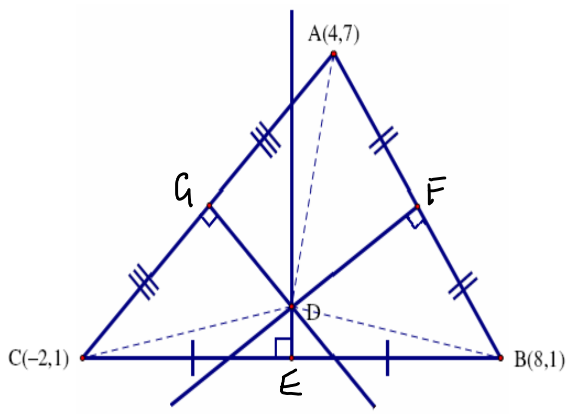
$$y - b = m(x - a)$$

$$y - 2 = \frac{2}{9}(x - 11)$$

$$9y - 18 = 2x - 22$$

$2x - 9y - 4 = 0$ is equation of altitude FG.

5.



BC horizontal $\Rightarrow$ DE is $x = 3$

E(3,1) F(6,4) G(1,4)

$$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7-1}{4-(-2)}$$

$$= \frac{6}{6}$$

$$m_{AC} = 1$$

$\Rightarrow \quad m_{GD} = -1$ as $m_1 . m_2 = -1$

for $\perp$ lines

$$y - b = m(x - a)$$

$$y - 4 = -1(x - 1)$$

$$y - 4 = -x + 1$$

$$y = 5 - x$$

For points of intersection solve $y = 5 - x$ and $x = 3$

$$\Rightarrow \quad y = 5 - (3)$$

$$y = 2$$

$\Rightarrow$ circumcentre at $D(3,2)$